ELECTRICAL MODELS FOR THE SOLUTION OF THE NONLINEAR PROBLEMS OF STEADY HEAT CONDUCTION, WITH USE OF NONLINEAR RESISTANCES

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We propose the use of electron tubes in the modeling of nonlinear boundary conditions in the solution of problems of steady heat conduction by an electric-model method, thus making it possible to solve the most complex of problems on the simplest of analog computers.

The possibility of solving a nonlinear equation for steady heat conduction on electric models has been demonstrated in a number of references [1, 2, etc.]. Here, to achieve a solution, we employed the method of successive approximations which, in the general case, for each approximation requires the recalculation and respecification of either all of the resistances of an R network, or of the resistances simulating the boundary conditions.

A new method is proposed in [3] for the modeling of nonlinear boundary conditions with the aid of nonlinear electrical resistances (incandescent lamps and current regulators), thus making it possible to solve the problem in one step (without successive approximations).

This paper, a development of the above-cited method, eliminates one of its most significant drawbacks – the need to maintain a reserve of a large set of nonlinear elements.

With this purpose in mind, in the place of incandescent lamps we propose the use of electron tubes (triodes, beam tetrodes, pentodes, heptodes) whose initial segments of the anode characteristics represent a family of parabolic-type curves, as the nonlinear element modeling the nonlinearity of the left-hand member of the boundary condition

$$\alpha\left(\sqrt{\theta_M}-\sqrt{\theta_f}\right)=-\frac{1}{h}\left(\theta_M-\theta_N\right). \tag{1}$$

The research that has been carried out demonstrated that by changing the bias voltage U_b at the control grid of the tube, and also by altering the resistance R_p, connected in parallel with the tube, we can achieve a nonlinear resistance with the function $I = f(U)$, close to a quadratic parabola $I = A\sqrt{U}$, where $A = f(U_b, R_p)$.

Since the identical tube can be used to approximate quadratic parabolas exhibiting various values for the coefficient A, it becomes a universal nonlinear element, because the identical tube can also be used to solve various problems. Fitting the characteristics of the nonlinear resistance to the specified quadratic parabola is accomplished with a specially developed unit by matching (on the screen of an oscillograph) the derived curve $I = A_S\sqrt{U}$ and that of the standard parabola $I = f(U)$ that has been plotted on the screen. The gain for the vertical-deflection amplifier of the oscillograph is calibrated and set equal to A/A_S.

To model the second term in the left-hand member of Eq. (1), which in the steady-state problem is a constant quantity, we can either use a current regulator, as in [3], or a controlled current stabilizer with a large internal resistance whose output current is proportional to the voltage applied to its input. This last simulation method seems preferable to us, since it frees us of the need to be dependent on the presence of current regulators and since it permits greater freedom of action when complex boundary conditions such as (1) also prevail at other boundaries of this region.

If the boundary condition in the electrical model is to be realized as in Fig. 1, for the point M we can write the Kirchhoff law

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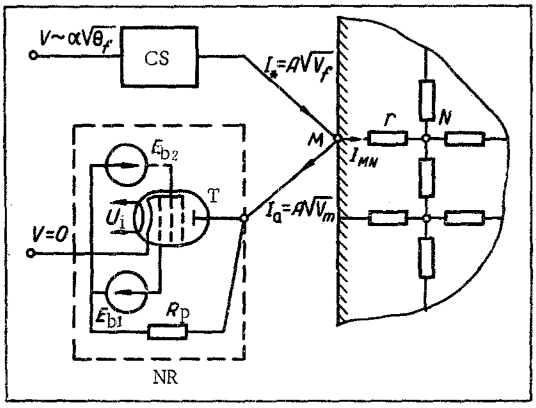

Fig. 1

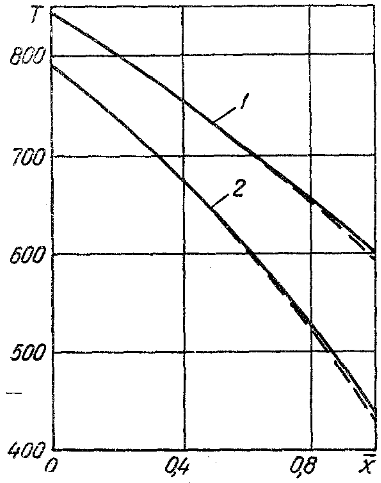

Fig. 2

Fig. 1. Diagram for the specification of the nonlinear boundary conditions on electrical models.

Fig. 2. Comparison of the results from the modeling with the analytical solution (the dashed lines show the results from an analytical calculation).

$$I_* + I_a + I_{MN} = 0$$

or [3]

$$A\left(\sqrt{V_M} - \sqrt{V_f}\right) = -\frac{1}{r}\left(V_M - V_N\right). \tag{2}$$

Comparing (1) and (2), it is easy to prove that they are identical, if we satisfy the criterial relationship derived in [3]:

$$\frac{Ar}{\alpha h}\sqrt{\frac{\theta_f}{V_f}} = 1. \tag{3}$$

This equation, in conjunction with the expressions for the scale factors and the function $I_* = f(V_f)$, is used to calculate the parameters of the electrical model.

To test this method with the use of electron tubes as the nonlinear elements, we solved the problem of steady-state heat conduction for an infinite plate, and the exact solution of this problem is given in [4]. Boundary conditions of the third kind are set up on both boundaries of the plate, these conditions varying in the different versions in terms of the heat-transfer coefficients on the side at which the heat flows in and at the side at which the heat flows out. We used 6Zh3P pentodes as the nonlinear resistances.

Figure 2 shows the temperature distribution in the plate for two combinations of the boundary conditions: $\alpha_{in} = \alpha_{out} = 232.6$ W/m^2 · deg (curve 1) and $\alpha_{in} = 232.6$ W/m^2 · deg; $\alpha_{out} = 885$ W/cm^2 · deg (curve 2). The temperature of the heating medium is 1073°K, and that of the cooling medium is 373°K. Table 1 shows the parameters of the electrical model for both cases of boundary-condition specification (for 1 and 2, see Fig. 2). Comparison of the modeling results with the data of the analytical calculation (Fig. 2) bears out the adequate accuracy of the derived solution (the error does not exceed 2-3%).

TABLE 1. Parameters of the Electrical Model

Parameters	1		2	
	heat inflow	heat outflow	heat inflow	heat outflow
I_*, mA	1	0,46	1	1,76
$A \cdot 10^4$	2,88	2,88	2,88	10,97
U_{p_1}, V	−1,9	−1,9	−1,9	−1,2
U_{p_2}, V	100	100	100	100
R_n, kΩ	27	27	27	11
V_f, V	12,06	2,57	12,06	2,57

Thus the proposed method, since it does not require linearization of the boundary conditions, since it permits solution of the problem in a single step, without successive approximations, and since it uses the simplest of models for the solution – networks assembled of constant resistances, i. e., exhibiting all of the advantages of the method described in [3] – significantly simplifies this last method, replacing the large assortment of nonlinear elements with a limited number of electron tubes, thus making the model universal and suitable for the solution of a variety of problems.

In conclusion, it should be stated that the use of electron tubes as the nonlinear elements expands the potentials of the method of nonlinear resistances on the whole, since it becomes possible to generalize this method to the case of an arbitrary relationship between the coefficient of thermal conductivity and temperature.

NOTATION

$\theta = \lambda^2$;

$\lambda = (a + bT)$	is the coefficient of thermal conductivity;
T	is the temperature of material;
T_f	is the temperature of the medium;
α	is the heat-transfer coefficient;
h	is the grid pitch;
I	is the current;
U	is the voltage;
V	is the potential;
R_p	is a resistance connected in parallel to the tube;
r	is the resistance which corresponds to the half-pitch of the grid;
U_{b1} and U_{b2}	are the bias voltages at the corresponding tube grids;
I_a	is the anode current;
$I_* = A\sqrt{V_f}$;	
I_{MN}	is the current between points M and N;
θ_f and V_f	are the values of the function θ and of the potential, corresponding to T_f;
CS	is the current stabilizer;
NR	is the nonlinear resistance;
T	is the tube;
U_i	is the incandescence;
E_{b1} and E_{b2}	are the bias sources;
$\bar{x}$	is the relative width of the plate;
a, b, and A	are proportionality factors.

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